



Cosmic Calibration - Statistical Modeling for Dark Energy

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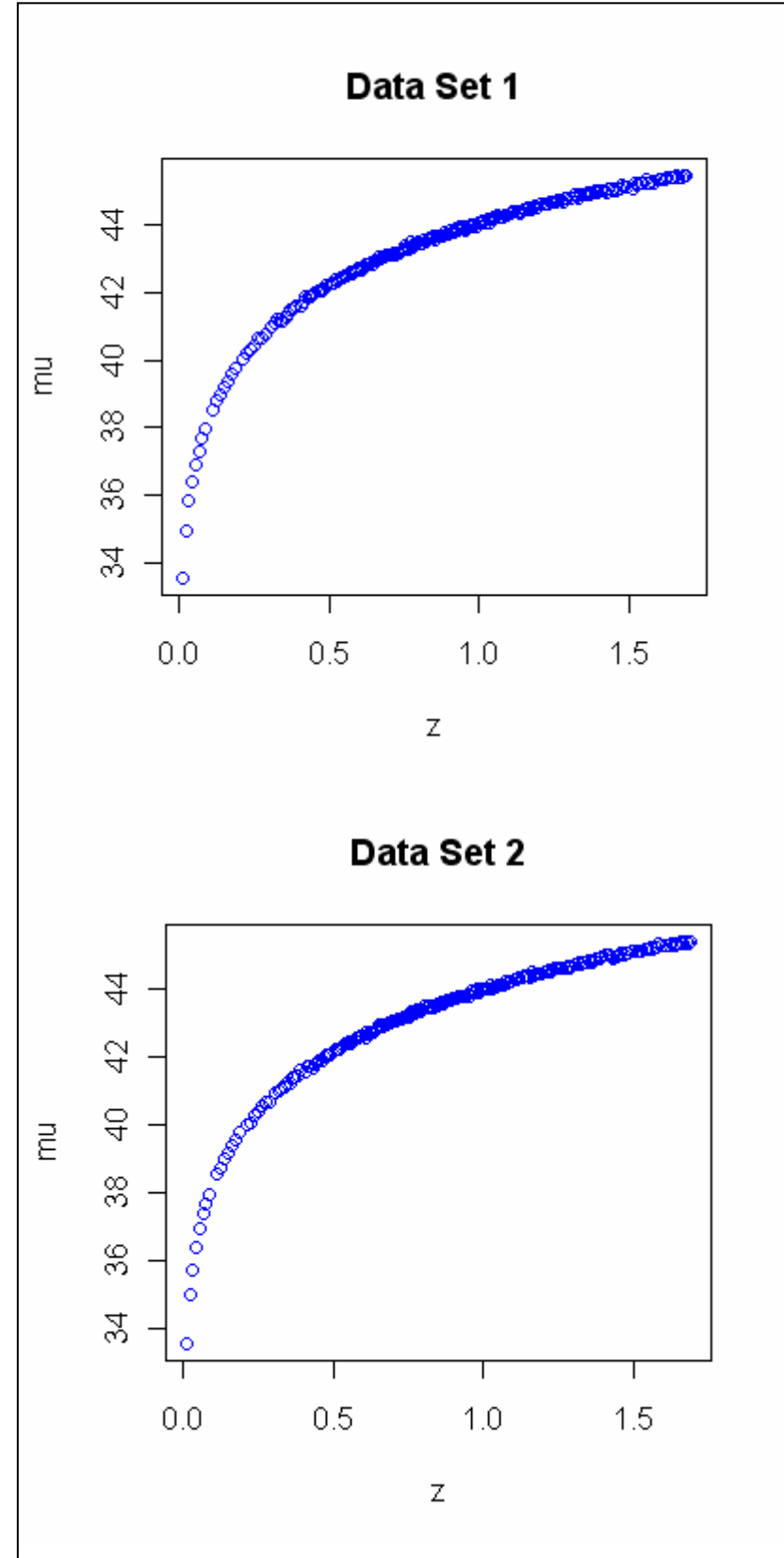


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Abstract

The fact that the Universe is expanding has been known since the 1920's. If the Universe was filled with ordinary matter, the expansion should be decelerating. Beginning in 1998, however, observational evidence has been accumulating in favor of an accelerating expansion of the Universe. The unknown driver of the acceleration has been termed dark energy. The nature of dark energy can be investigated by studying its equation of state, that is the relationship of its pressure to its density. The equation of state can be measured via a study of the luminosity distance-redshift relation for supernovae. In this study, we employ supernovae data, including measurement errors, to determine whether the equation of state is constant or not. Our method is based on Bayesian analysis of a differential equation and modeling $w(z)$ directly, where $w(z)$ is the equation of state parameter. This work stems from collaboration between UCSC and Los Alamos National Laboratory (LANL) in the context of the Institute for Scalable Scientific Data Management (ISSDM) project.

Original Data



These two data sets are simulated data sets where we know the truth. Both data sets have 215 observations of supernovae observations (SNe Ia.). The data has a redshift (z) value for each supernova and a value for μ (observed distance modulus.) The first plot is of z vs μ .

Because the data sets are simulated we have values for $\Omega_m = 0.27$ and $H_0 = 72.0$. In data set 1, $w(u) = -1$ and in data set 2, $w(u)$ is a non-linear curve. It is impossible to see the difference in $w(u)$ in these plots because it is just one parameter of the model.

We will look specifically at two models to fit these data sets. The first will be a parametric model that has been studied in depth and the other is a Gaussian Process model. Both of these models will be implemented using Bayesian methods and MCMC algorithms.

Equations and Parameters of Interest

The main parameter of interest is $w(u)$ there are also two other known parameters: $H_0 = 72.0$ and $\Omega_{m0} = 0.27$. Where the uncertainty shown is one standard deviation. The main equation of interest is a transformation:

$$T(z, H_0, \Omega_m) = 25 + 5 \log_{10} \left(\frac{c(1+z)}{H_0} \int_0^z \left(\Omega_m (1+s)^3 + (1-\Omega_m)(1+s)^3 e^{\int_0^s \frac{w(u)}{1+u} du} \right)^{-0.5} ds \right)$$

To be able to use this equation we will need to specify a form for $w(z)$. This also leads to a likelihood as follows:

$$L(\sigma, w_0, H_0, \Omega_m) \propto \left(\frac{1}{\tau_i \sigma} \right)^n e^{-\frac{1}{2} \sum_{i=1}^n \left(\frac{\mu_i - T(z_i, H_0, \Omega_m)}{\tau_i \sigma} \right)^2}$$

To be able to use this likelihood we will need priors for σ and whatever parameters we used to specify $w(u)$. As a note the τ 's are the standard deviations for μ and part of the data set.

Parametric Model Theory

This non-linear model is advocated by Linder, a cosmologist, as a good alternative to $w(u)$ set equal to a constant or just a simple line.

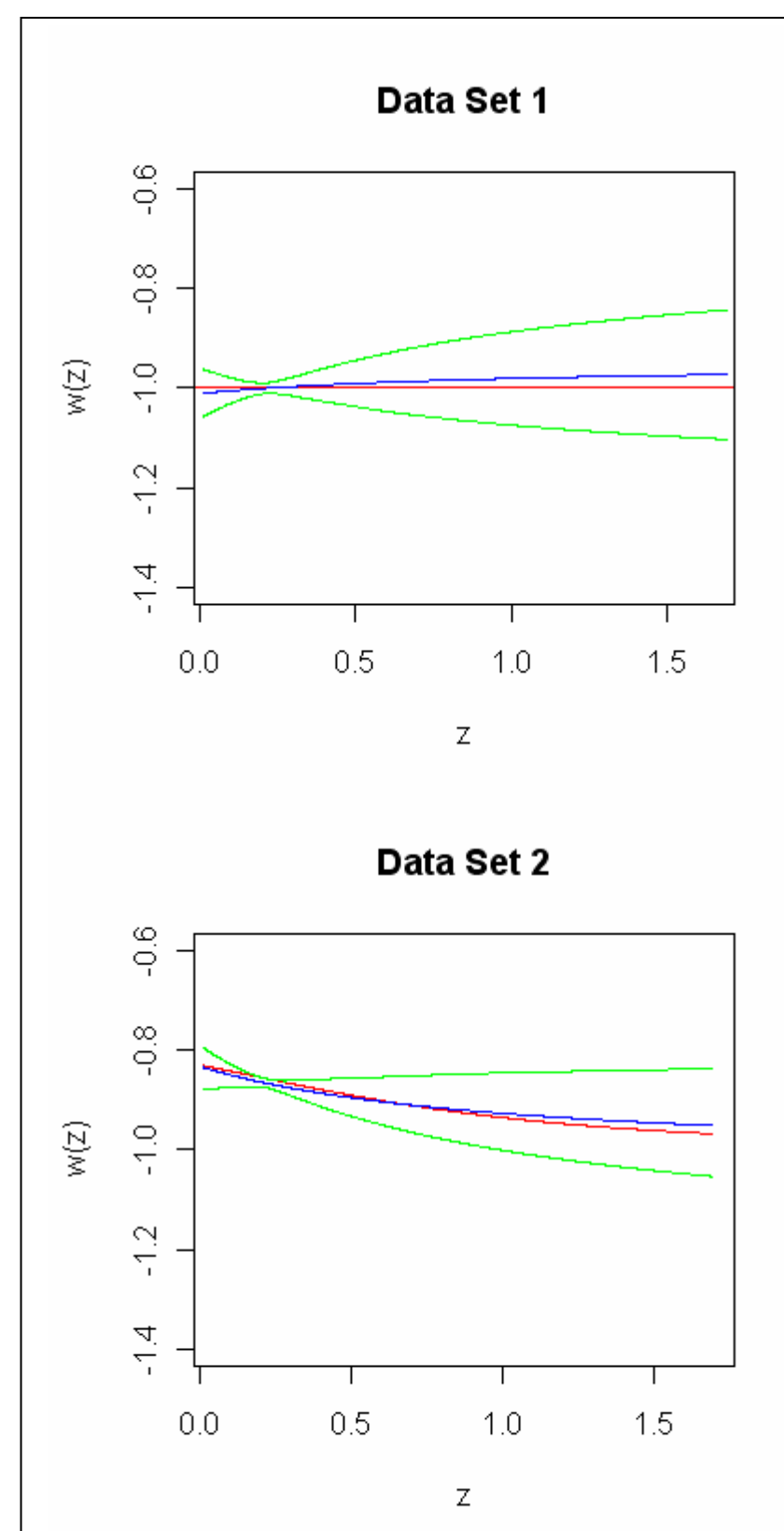
$$w(u) = a + b \left(\frac{1}{1+u} - 1 \right)$$

This leads to a simplified version of our equation, namely we were able to do one of the integrations analytically.

$$T(z, H_0, \Omega_m) = 25 + 5 \log_{10} \left(\frac{c(1+z)}{H_0} \int_0^z \left(\Omega_m (1+s)^3 + (1-\Omega_m)(1+s)^{3(a-b+1)} e^{\frac{3bs}{1+s}} \right)^{-0.5} ds \right)$$

To be able to use this likelihood we will need priors: $\pi(a) \sim U(-20,0)$, $\pi(b) \sim U(-20,0)$, and $\pi(\sigma^2) \sim \text{IG}(10,9)$. We will use this model to compare against our Gaussian Process model.

Parametric Model Results



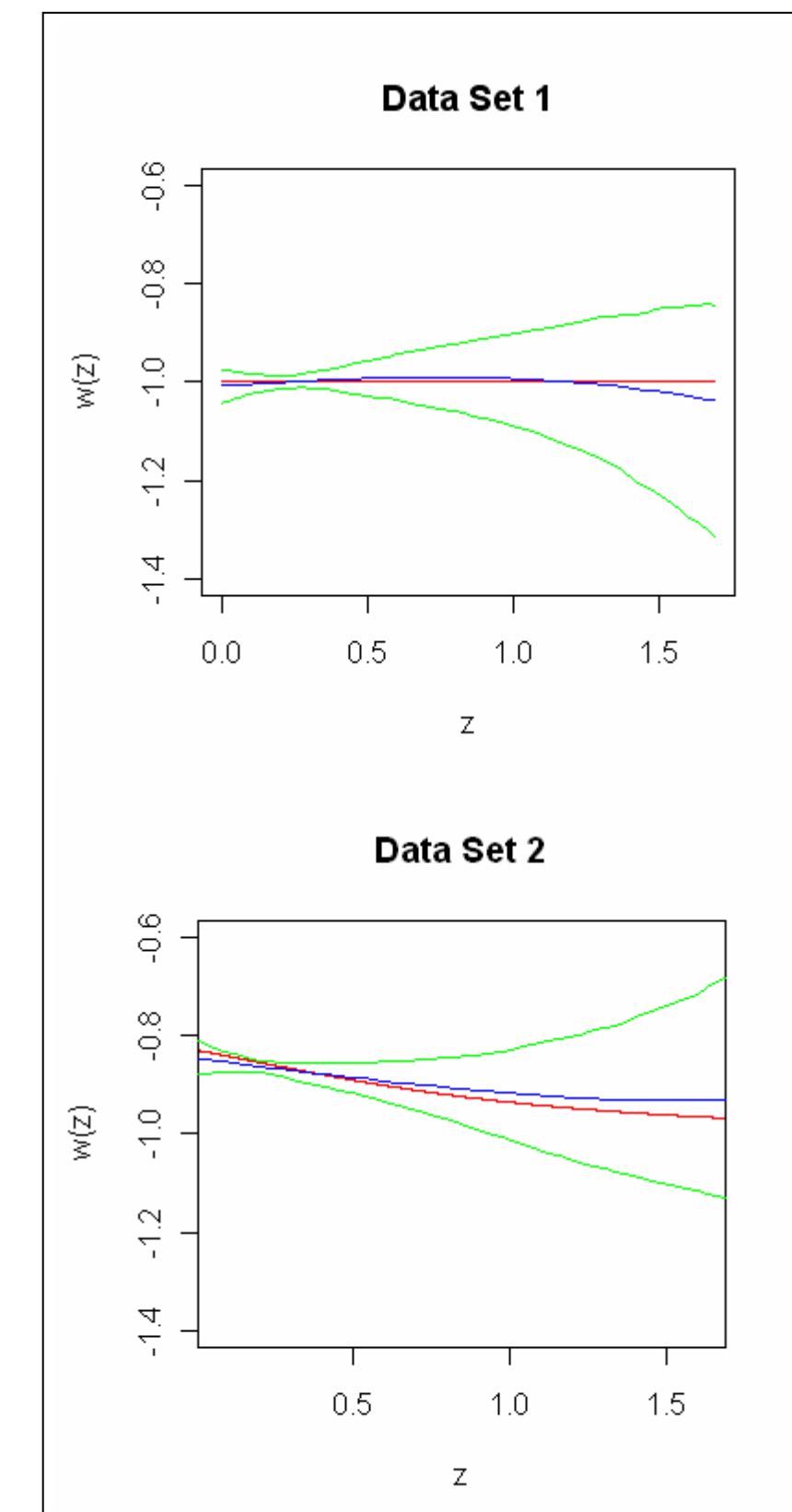
The red line is the truth; we know it for this case because the data sets are simulated.. For the data set 1 (top) we have $w(u) = -1$ and for data set 2 (bottom) $w(u)$ is non-linear. The blue line is the mean fit of the MCMC simulation and the green lines are the 95% probability intervals.

Overall, model three is able to closely fit the "truth." We are also able to add more unknown parameters to this model like H_0 and Ω_m . When adding more parameters the probability bands get much larger but still include the truth. It should be noted that in the MCMC that the parameters are correlated and drawn jointly.

Table 1 - Posteriors for the Three Parameters

| | Data set 1 | Data set 2 |
|---|---------------|---------------|
| 95% Probability interval for a | (-1.06,-0.96) | (-0.88,-0.79) |
| 95% Probability interval for b | (-0.34,0.22) | (-0.04,0.43) |
| 95% Probability Interval for σ^2 | (0.36,0.50) | (0.32,0.43) |

Gaussian Process Model Results



The red line is the truth; we know it for this case because the data sets are simulated.. For the data set 1 (top) we have $w(u) = -1$ and for data set 2 (bottom) $w(u)$ is non-linear. The blue line is the mean fit of the MCMC simulation and the green lines are the 95% probability intervals.

The GP is capturing the "truth" but it has wider probability bands because we do not specify a parametric form. We also were not able to add more unknown parameters to this model like H_0 and Ω_m . The mixing is slow for the GP and it must be run many iterations. It should be noted that in the MCMC that the parameters are correlated and drawn jointly.

Table 1 - Posteriors for the Three Parameters

| | Data set 1 | Data set 2 |
|---|---------------|---------------|
| 95% Probability interval for ρ | (0.729,0.997) | (0.656,0.991) |
| 95% Probability interval for κ^2 | (0.25,0.50) | (0.25,0.50) |
| 95% Probability Interval for σ^2 | (0.36,0.50) | (0.32,0.44) |

Conclusions

- We have shown that a typical parametric model, as well as, a non-parametric Gaussian Process model can be fitted to the equation of state.
- The benefit of using a GP model is that it allows us to fit the equation of state without specifying a parametric form, which at this time is unknown.
- The GP produces larger probability bands because it is a more flexible model
- The GP is flexible enough to fit a curve somewhat away from the GP's mean of -1.

Future Work

- Use an orthogonal basis of damped Hermite polynomials to approximate $w(u)$
- Analyze other simulated data sets and real data sets using these methods
- Set up an experimental design to find where more data is needed (on the z axis). In the experimental design also test how shrinking uncertainty for μ , Ω_{m0} , and H_0 would help in drawing more conclusive statements about $w(z)$.
- Look into which type of measurement error could be reduced to help make conclusive statements about the parameters of interest; especially the standard deviations associated with μ

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